NOVEL MATRIX ALGORITHM for VLSI INTERCONNECTS MODEL-ORDER REDUCTION

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Abstract- -The algorithm of VLSI interconnects model-order reduction is offered. It allows on the basis of general matrix of the circuit to form their macromodels equations as two-ports with subsequent their schematic interpretation for providing of possibility of the use of the existent simulation programs.

1.Introduction

As is generally known, interconnections in the modern VLSI chips are occupied greater part of their area (to 70%), bringing in determining contribution to the delays of signals at their transmission from a block to the block of the circuit. At the design these interconnections are considered as transmission lines and usually replaced by equivalent circuits with the lamped RLC-components having the huge number of internal nodes of connecting, that does the general VLSI model is unacceptably large for existent circuit simulation software.

Today there are two main approaches to extracted parasitic interconnect data size decreasing through the model order reduction technique:

- generating stable and passive macromodels (the AWE [1], RICE [2], Arnoldi [3], PRIMA algorithms [4]);
- Y-Δ transforming (the TICER [5] algorithm and others [6-9]).

The macromodels being developed under the first approach cannot usually directly be fed into a general simulator and they are need some previous schematic interpretation. That why RC-in RC-out (or LRC-in-LRC) reduction scheme has been successfully incorporated into design procedure (for example, TICER) by eliminating internal nodes with minimal loss in accuracy and with extremely large and small time constants. Such direct reducing an extracted netlist avoids the modification of existing simulation tools. But it is understandable that for dense and large circuit topologies where each node has several neighbors this approach performs poorly.

From mathematic point of view Y/Δ transformation resembles inner nodes Gauss eliminating in the general circuit admittance matrix Y :

$$y^*_{jk} = y_{jk} - \frac{y_{ji} \cdot y_{ik}}{Y_{ii}}, \qquad (1)$$

where y_{ii} - a matrix diagonal element.

It shows that even in the case when there is not the element between node *j* and *k* $(y_{jk} = 0)$ a new element $y^*_{jk} \neq 0$ will appear if the node *i* is eliminated.

The existing macromodels approaches use moment matching and projection techniques to generate a low order approximation of the original circuits as two-port and multi-port LRC circuits. These methods can't work with circuit matrix *Y* directly and needs *the state equations* of the circuit [4]:

$$C \cdot x' = G \cdot x + B^t \cdot u,$$
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = L \cdot u, \qquad (2)$$

where C, G, B and L- matrices of equation (2), x – state variables vector,

$u = [u_1 \ u_2]^t$ и $i = [i_1 \ i_2]^t$ – port voltages and currents which coincide with ports of the two-port macromodels

Two-port macromodel matrix is defined by

$$Y^p_{tp} = M_o + s \cdot M_1 + s^2 \cdot M_2 + s^3 \cdot M_3 + ... \quad (3)$$

where $M_i = L \cdot A^j \cdot R, \quad j = 0,1,2,...$

- the moments of the original circuit being computed through matrices of equation (2)

$A = -G^{-1} \cdot C, \quad R = G^{-1} \cdot B^t$.

In the article an alternative macromodels technique based on two-port circuit (or N-port in general case) being derived directly from the general nodal equations (or modified nodal equations) of the original circuit is presented. The offered algorithm is far more effective from point of calculable expenditures by comparison to the PRIMA algorithm based on expressions (3) – (4) realization. In addition, procedure of schematic interpretation of the two-port equations is proposed , which jointly with the procedure of optimization allows to build simple schematic macromodels of VLSI interconnects with enough high exactness.

II. Alternative macromodels method

The two-port macromodel can be written as the matrix of transfer admittances between every couple of ports of the reduced two-port in the form:

$$\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} Y_{aa} & Y_{ab} \\ Y_{ba} & Y_{bb} \end{bmatrix} \begin{bmatrix} U_a \\ U_b \end{bmatrix}, \qquad (4)$$

where I_a ,U_a –the current into port 1 and the voltage across port 1; I_b , U_b –the current into port 2 and the voltage across port 2.

Exact transfer admittances matrix can be also realized without circuit moments calculating by using another approach and with lesser computer expenditure. A general idea is to obtain the y-parameters of the reduced multi-port by the direct computation of the matrix inverse to the initial matrix of the given passive RLC-circuit. Assume that the inverse matrix $\mathbf{Y}^{-1}$ is obtained for any fixed frequency ω_0:

$$Y^{-1} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{21} \cdots & \Delta_{n1} \\ \Delta_{12} & \Delta_{22} \cdots & \Delta_{n2} \\ & & \\ \Delta_{1n} & \Delta_{2n} \cdots & \Delta_{nn} \end{bmatrix}. , \qquad (5)$$

where Δ is the determinant of the initial matrix **Y** and Δ_{ij} is the cofactor of the Y matrix obtained by the deleting the i-th row and j-th column in matrix **Y**.

The transfer admittance $Y_{i,j}(i)$ from the <a> port to the <b> port can be calculated using the inverse matrix $\mathbf{Y}^{-1}(\mathbf{s})$ elements as follows [10]:

$$Y_{tp}(i) = \frac{1}{\Delta_{aa,bb}} \cdot \begin{bmatrix} \Delta_{bb} & -\Delta_{ba} \\ \Delta_{ab} & -\Delta_{aa} \end{bmatrix}. \qquad (6)$$

The doubled cofactor which is in the denominator can be obtained by the deletion of two rows and two columns in the initial Y matrix. For instance, $\Delta_{aa,bb}$ is the determinant of the initial Y matrix in which rows a and b are deleted as well as columns a and b. The sign of this minor is $(-1)^{a+a+b+b}$.

It is significant to note that really there is no need to calculate as this doubled cofactor so other cofactors Δ_{ij} ! They can be obtained through the elements of the inverse matrix $\mathbf{Y}^{-1}$,for example,

$$\Delta_{aa,bb} = \frac{\Delta_{bb}\Delta_{aa} - \Delta_{ba}\Delta_{ab}}{\Delta} , \qquad (7)$$

where Δ is the determinant of the initial matrix **Y**.

In case of using the inverse matrix $\mathbf{Y}^{-1}$ elements we get the numerical values of the macromodel y-parameters $Y_{i,j}(s)$ as the sum of the real $Y_{(i,j)0}$ and imaginary $Y_{(i,j)s}$ components of $Y_{i,j}(s)$, then the reduced model of the RLC-multi-port at frequency ω_0 can be represented in the form:

$$Y_{tp} = \begin{bmatrix} Y_{aa0} & Y_{ab0} \\ Y_{ba0} & Y_{bb0} \end{bmatrix} + s \cdot \begin{bmatrix} Y_{aas} & Y_{abs} \\ Y_{bas} & Y_{bbs} \end{bmatrix}. \qquad (8)$$

It is worth to mention that in the case of two-port with *a*-port and *b*-port it is necessary to select four elements from elements of inverse matrix $\mathbf{Y}^{-1}$:

$$Y^{-1} = \begin{bmatrix} g_{11} & g_{21} \cdots & g_{n1} \\ g_{12} & g_{22} \cdots & g_{n2} \\ & & \\ g_{1n} & g_{2n} \cdots & g_{nn} \end{bmatrix}. , \qquad (9)$$

These elements are: $g_{aa}, g_{bb}, g_{ab}, g_{ba}$, from which it is possible to calculate

$$g_{aa,bb} = g_{aa} g_{bb} - g_{ab}, g_{ba}. \qquad (10)$$

The *y*-parameters of reduced circuit model are equal to:

$$Y_{aa} = g_{bb} / g_{aa,bb};$$
$$Y_{ab} = -Y_{ba} = -g_{ba} / g_{aa,bb}; \qquad (11)$$
$$Y_{bb} = g_{aa} / g_{aa,bb}.$$

Possibility is kept also to write down the parameters of macromodel in the form of fractional-rational transfer functions from complex frequency. For this purpose it is necessary to go back to expressions (6) and (8), and additionally to finding elements g_{ij} of inverse matrix to calculate the determinant of matrix Δ and necessary algebraic cofactors $\Delta_{ij} = g_{ij} \cdot \Delta$. Calculation of expressions (11) on different frequencies confirms that for linear circuits, at least, for frequencies $\omega \le \omega_0$, where ω_0 is the frequency on which the elements of inverse matrix (9)are originally determined, real parts of the calculated parameters (11) are practically saved, and imaginary parts change to the proportionally chosen frequency. It confirms of principal possibility of application of parameters of schematic macromodels calculated on one frequency in a wide frequency range.

III.Schematic presentation of the reduced model of the RLC-multi-port

Since, the numerical values of the y-parameters $Y_{ij}(s)$ in the formulas (4) and (8) are, generally, the sum of the real and imaginary components of $Y_{ij}(s)$:

$$Y_{ij} = a_o + s\, a_1 .$$

Then in case of $a_o > 0,\ a_1 > 0$ the reduced model of the RLC-multi-port at frequency ω_0 can be represented in the form of two components (resistor and capacitor) connected in parallel (see Fig.1) if these components are both positive.

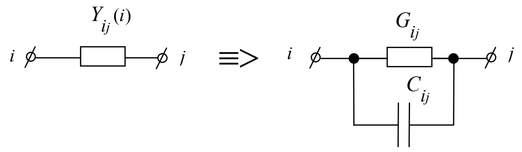

Fig.1. Transfer admittance representation in the form of the circuit of the two components

The such simplest schematic interpretation of macromodel equations (8) results in appearance of the resistances connected between nodes *a* and *b* and earth in schematic macromodel that distorts an amplitude of output signal.
Therefore expediently to choose schematic circuit macromodel priory, for example, shown on fig.2 and to find links between the parameters of its components and parameters of mathematical macromodel (8).

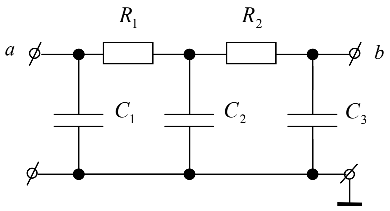

Fig.2. Possible macromodel equivalent circuit

For example for the reduced circuit model shown on fig.2 we have got the following real and image part of two-port *y*-parameters:

$$Y_{aa0} = -\frac{g_1^2(g_1+g_2)}{(g_1+g_2)^2+(\omega C_2)^2} + g_1;\quad Y_{aas} = \frac{g_1^2\omega C_2}{(g_1+g_2)^2+(\omega C_2)^2} + \omega C_1;$$

$$Y_{ab0} = \frac{g_1(g_1+g_2)g_2}{(g_1+g_2)^2+(\omega C_2)^2};\quad Y_{abs} = \frac{g_1\omega C_2 g_2}{(g_1+g_2)^2+(\omega C_2)^2};\qquad (11)$$

$$Y_{bb0} = -\frac{g_2^2(g_1+g_2)}{(g_1+g_2)^2+(\omega C_2)^2} + g_2;\quad Y_{bbs} = \frac{g_2^2\omega C_2}{(g_1+g_2)^2+(\omega C_2)^2} + \omega C_3,$$

From above relations it is possible to calculate circuit components parameters:

$$g_2 = \frac{Y_{bb0}\cdot Y_{aa0} - (Y_{ab0})^2}{Y_{ab0}+Y_{aa0}};\quad g_1 = \frac{Y_{bb0}\cdot Y_{aa0} - (Y_{ab0})^2}{Y_{ab0}+Y_{bb0}};$$

$$\omega C_1 = Y_{aas} - Y_{abs}\frac{g_1}{g_2};\quad \omega C_3 = Y_{bbs} - Y_{abs}\frac{g_2}{g_1};$$

$$\omega C_2 = (g_1+g_2)\cdot\frac{Y_{abs}}{Y_{ab0}}. \qquad (12)$$

To be true it is necessary to notice that we have defined 5 unknown parameters from 6 equations (11) that makes some uncertainness and needs for exact solution to apply sometimes optimization procedure to meet prescribed design parameters.

IY. Algorithm modification for a huge dimension of the Y matrix of the initial RLC-multi-port

Because of their structural regularity RLC-circuits with lamped parameters can be represented in the form of relatively low-order multi-ports connected in series (something like the schematic shown in Fig.3). By the way, all the example circuits given in the referred original papers are of a regular structure.

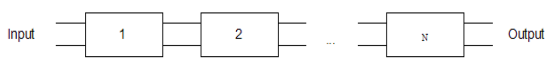

Fig.3. Representation of the initial high-order RLC-two-port with lamped parameters in the form of N low-order two-ports with lumped parameters

This leads one to represent the entire multi-port A matrix in the form of the product of the A matrices of two-ports

$$A_N = A_1 A_2 \ldots A_S \qquad (13)$$

Let remind that the A matrix elements of any two-port relate the output and input voltages and currents as follows ,

$$\begin{bmatrix} U_a \\ I_a \end{bmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} U_b \\ I_b \end{bmatrix}$$

where subscripts “*a*” and “*b*” denote the input and output voltages *U* and currents *I* of input and output ports.

Another useful property of elements of the A matrix is that they have one-to-one correspondence with the elements of the Y matrix of the same two-port:

$$\begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} = \frac{1}{Y_{ba}} \begin{bmatrix} -Y_{bb} & 1 \\ -|Y| & Y_{aa} \end{bmatrix} \qquad (14)$$

and

$$\begin{bmatrix} Y_{aa} & Y_{ab} \\ Y_{ba} & Y_{bb} \end{bmatrix} = \frac{1}{A_{ab}} \begin{bmatrix} A_{bb} & -|A| \\ 1 & -A_{aa} \end{bmatrix}, \qquad (15)$$

where $|A|, |Y|$ - determinants of matrices of equations (16) and (5) accordingly. It should be noted that for passive linear RLC-charts the determinant $|A| = 1$, that ensues from (15), because $\frac{-Y_{bb}Y_{aa} + |Y|}{Y_{ba}^2} = 1$ at $Y_{ba} = -Y_{ab}$. This statement is possible to use for verification of rightness of calculation of A–parameters of a macromodel.
From another side elements of the A matrix of the two-port may be computed directly from cofactors of the Y matrix of the original circuit or from elements of its inverse matrix [10]

$$A_{tp}(i) = \frac{1}{\Delta_{ab}} \begin{bmatrix} \Delta_{aa} & \Delta_{aa,bb} \\ \Delta & \Delta_{bb} \end{bmatrix} \qquad (16)$$

and

$$A_{tp}(i) = \frac{1}{g_{ab}} \begin{bmatrix} g_{aa} & g_{aa,bb} \\ 1 & g_{bb} \end{bmatrix}, \qquad (17)$$

or $A_{aa} = g_{aa}/g_{ab}$. $A_{ab} = g_{aa,bb}/g_{ab}$; $A_{ba} = 1/g_{ab}$; $A_{bb} = g_{bb}/g_{ab}$, (18)

where as before $g_{aa,bb} = g_{aa}g_{bb} - g_{ab}, g_{ba}$.
Using the properties of A and Y matrices, we can then recalculate the A-matrix parameters into the Y-matrix parameters.
So the following sequence of computations can be proposed:
1.The initial large-scale RLC-circuit is divided into N relatively simple small-scale subcircuits (each with 10 – 20 internal nodes). The procedure of calculation of a large-scale inverse matrix Y^{-1} can be reduced to the calculation of N inverse matrices Y^{-1}_1, Y^{-1}_2,, Y^{-1}_N for the subcircuits connected in series.

2.Using the elements of inverse matrices, the *y*-parameters or *a*-parameters of each subcircuit can be formed according to relations (11) and (15).
3.The A_N matrix of the entire circuit is calculated in according to (13) by the multiplication of matrices A_i
4.By elementary transformations (15) the elements of the Y(s) matrix of the reduced RLC-multi-port are obtained.
5.Obtained two-port *y*-parameters are used for macromodel circuit presentation according to formulas (12), using the real and the imaginary parts of each y-parameter.

Y. Design example

Let consider the first test circuit being used by Intel and NTUU"KPI" (fig.4) , for which delay time is D = 0.121820654E-08 sec

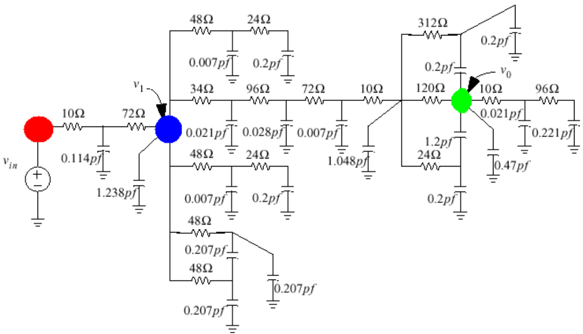

Fig.4. First RLC testing circuit

Using the approach being described above we have got after optimization correction the following results:

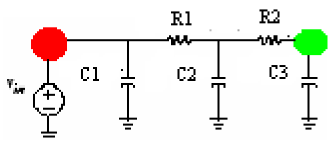

Fig.5. Developed macromodel (R1 = 81.999999Ω; C1= 0.08789054133 pF; R2= 64.117 Ω ; C2 = 10.918 pF ; C3 = 0.6440630558 pF)

Proposed macromodel possesses the time delay D = 0 .122118380E-02 usec (accuracy 0.244 %) when KPI solution (based on Y/Δ transformation) is D = 0 .118828495E-02 usec (accuracy 2.45%).
For second Intel test circuit, being shown in the fig.6 macromodel was developed also/

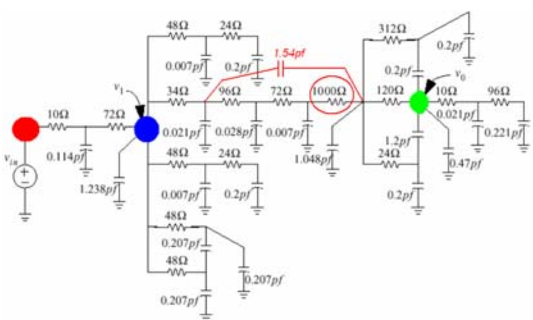

Fig 6. Second RLC test scheme

For this macromodel (fig.7) where C2=0 delay time is D = 0 .177271245E-02 usec (accuracy 0.17%) when original circuit (fig.6) has D = 0 .176957782E-02 usec;

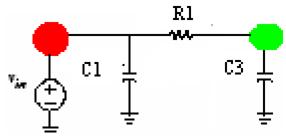

Fig.7. Macromodel with C2=0 (R1 = 1403.963499 Ω; C1 = 0.08789056438 pF; C3= 1.1989 pF)

Conclusions

The proposed two-port macromodel approach seems to be more simple and effective in comparison with existing methods particularly PRIMA.
It allows in principal to deals with the very large circuit using subcircuits technique.
The two-port matrix parameters can be corrected through the optimization during simulation procedure to get identical design parameters with the original circuit. This also avoids numerical mistakes of previous calculations.
Schematic representation of the two-port is restricted to-day only to some simple cases and it is need to be investigated more attentively.

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